Numerical Solution of Weakly Regular Volterra Integral Equations of the 1st Kind

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We propose new numerical method for solution of the following linear scalar Volterra integral equation of the 1st kind

$$\int_0^t K(t,s)x(s)ds = f(t), \ \ 0 \le s \le t \le T, \ \ f(0) = 0,$$

where kernel is defined as follows

$$K(t,s) := \begin{cases} K_1(t,s), \ t,s \in m_1, \\ \ \ \dots \ \ \dots\dots\dots \\ K_n(t,s), \ t,s \in m_n, \end{cases} \quad \begin{array}{l} m_i := \{t,s \mid \alpha_{i-1}(t) < s < \alpha_i(t)\}, \\ \alpha_0(t) = 0, \ \alpha_n(t) = t, \ i = \overline{1,n}, \end{array}$$

$\alpha_i(t), \ f(t) \in \mathcal{C}^1_{[0,T]}$, $K_i(t,s)$ have continuous derivatives (w.r.t. t) for $t,s \in \overline{m_i}$, $K_n(t,t) \neq 0$, $\alpha_i(0) = 0, \ \ 0 < \alpha_1(t) < \alpha_2(t) < \dots < \alpha_{n-1}(t) < t$, $\alpha_1(t), \dots, \alpha_{n-1}(t)$ increase at least in the small neighborhood $0 \le t \le \tau$, $0 < \alpha'_1(0) \le \dots \le \alpha'_{n-1}(0) < 1$. The mid-rectangular quadrature rule is employed for the numerical method construction. The accuracy of proposed numerical method is $\mathcal{O}(1/N)$. We introduce the following uniform mesh $\Omega_x^N := \{t_i | t_i = i/N, i = 0, \dots, N\}$, the mesh can be non-uniform $0 = t_0 < t_1 < t_2 < \dots < t_N = T, \ \ h = \max_{i=\overline{1,N}}(t_i - t_{i-1}) = \mathcal{O}(N^{-1})$ and seek the approximate solution $x_N(t) = \sum_{i=1}^{N} x_i \delta_i(t), \ t \in (0,T], \ \delta_i(t) = \begin{cases} 1, & \text{for } t \in \Delta_i = (t_{i-1}, t_i] \\ 0, & \text{for } t \notin \Delta_i \end{cases}$, where the coefficients $x_i, \ i = \overline{1,N}$ are under determination. Let us consider the equation $\int_0^{t/3} (1+t-s)x(s)\,ds - \int_{t/3}^{t} x(s)\,ds = \frac{t^4}{108} - \frac{25t^3}{81}, \ t \in [0, 2]$, where $\bar{x}(t) = t^2$ is exact solution. Table below demonstrates the errors $\varepsilon_N = ||x^N(t_i) - \bar{x}(t_i)||_{\Omega^N}$ and order of convergence $p^N = \log_2 \frac{D^N}{D^{2N}}$ based on maximum pointwise two-mesh differences $D^N = ||x^N(t_i) - \bar{x}^{2N}(t_i)||_{\Omega^N}$ without *a priori* knowledge of exact solition.

	32	64	128	256	512	1024	2048	4096
ε_N	0.13034	0.07804	0.03989	0.01975	0.01002	0.00508	0.00256	0.00128
D^N	0.07462	0.03815	0.02013	0.00975	0.00514	0.00259	0.00129	0.00065
p^N	0.96774	0.92207	1.04619	0.92217	0.98864	1.00716	0.98639	1.00198

This is joint work with Denis Sidorov and Alexander Tynda.

REFERENCES

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