SOLUTION OF LINEAR AND NONLINEAR VOLTERRA INTEGRAL EQUATIONS OF THE FIRST KIND WITH PIECEWISE SMOOTH KERNELS

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Volterra integral equations of the first kind $\int_0^t K(t,s)x(s)ds = f(t)$, $f(0)=0$, $0 \le s \le t \le T$, with piecewise smooth kernels

$$K(t,s) := \begin{cases} K_1(t,s), \ t,s \in m_1, \\ \ldots \ \ldots\ldots\ldots \\ K_n(t,s), \ t,s \in m_n, \end{cases} \qquad \begin{array}{l} m_i := \{t,s \mid \alpha_{i-1}(t) < s < \alpha_i(t)\}, \\ \alpha_0(t)=0, \ \alpha_n(t)=t, \ i=\overline{1,n} \end{array}$$

are considered. Here $\alpha_i(t)$, $f(t) \in \mathcal{C}^1_{[0,T]}$, $K_i(t,s)$ have continuous derivatives (w.r.t. t) for $t,s \in \overline{m_i}$, $K_n(t,t) \neq 0$, $\alpha_i(0)=0$, $\quad 0 < \alpha_1(t) < \alpha_2(t) < \cdots < \alpha_{n-1}(t) < t$, $\alpha_1(t), \ldots, \alpha_{n-1}(t)$ increase at least in the small neighborhood $0 \le t \le \tau$, $0 < \alpha_1'(0) \le \cdots \le \alpha_{n-1}'(0) < 1$. The following results will be reported for this equation: condition for the existence of a unique solution; conditions for the existence of parametric solutions. Based on these results the approximate methods (successive approximations and asymptotic methods) are constructed. Taking into account obtained conditions for the existence of a unique solution, the numerical methods are proposed in [4,5] and their first order convergence is observed. The theoretical results [1-3] are generalized on the cases of matrix kernels in [6] and operator coefficients $K_i(t,s)$ in [7]. The proposed theory can be employed for solutions of the problems in power systems development modeling, including optimal equipment replacement [5,8]. New results concering the generalizations on nonlinear case and piecewise smooth source function will be also reported.

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