RECOVERING IMAGES REGISTERED BY DEVICE WITH INEXACT HARDWARE FUNCTION USING TIKHONOV'S REGULARIZED LEAST SQUARES METHOD [1]

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The using Tikhonov's regularized least squares method (RLS) [1] to solve approximate systems of linear algebraic equations (SLAE) are considered. Said method is applied to solve image recovery problem for image, registered by device with inexact hardware function.

RLS Problem [1, 2, 3]. Let an exact consistent finite-dimensional SLAE $A_0x = b_0$ exists, where $A_0 \in R^{m\times n}$, $b_0 \in R^m$, $x_0 \in R^n$ — normal solution of said SLAE. Instead of A_0 and b_0, approximate matrix A and vector b are set so that to meet the constraints $\|A_0 - A\| \le \mu$, $\|b_0 - b\| \le \delta < \|b\|$. It is required to find such $A_1 \in R^{m\times n}$, $b_1 \in R^m$, and $x_1 \in R^n$ that $\|A_1 - A\| \le \mu$, $\|b_1 - b\| \le \delta$, $A_1x_1 = b_1$, $\|x_1\| \to \min$.

The common method for solving discussed problem is to solve Euler regularized system $\left(A^TA + \alpha I\right) x_\alpha = A^Tb$ with regularization parameter $\alpha > 0$.

As it shown in [2], there are such conditions that leads to necessity of using negative regularization parameter in Tikhonov's regularized least squares method. This case is interesting from both a theoretical and practical point of view, because its computing implementation requires special tools to overcome the bad numerical conditioning.

It is known (see, e.g. [4]), that the image recovery problem (when image was registered by device with inexact hardware function) may be reduced to problem of solving approximate SLAE. And approximate SLAE may be solved by RLS for certain values of regularization parameter.

The values of regularization parameter which provide minimal error of image recovering were found in computational experiments. The specially designed algorithms with minimal numerical conditioning were used for image recovering in that experiments. Said algorithms are working with any values of regularization parameter including $\alpha < 0$.

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