MODEL SHEET RECTANGULAR PACKING PROBLEM

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The paper considers the model of the sheet rectangular packing problem of the set of rectangles with a given sizes $a_i \times b_i$, $i = 1 \dots n$ $(a_i \geq b_i)$ on the sheet with given size $A \times B$. As the origin selects the lower-left corner of the sheet. Variables of the model are the coordinates placement rectangles on the sheet, x_i, y_i $i = 1 \dots n$, and three sets boolean variables: z_i $i = 1 \dots n$ (the orientation of i-th rectangle along or across a sheet), s_{ij}, t_{ij} $i = 1, \dots, n-1$ $j = i+1, \dots, n$ (used for the formulation of conditions of no intersections of ith and j-th rectangles on horizontal and vertical). The model is represented as the following system of linear inequalities with partial boolean variables:

$$x_i \geq 0, x_i + (b_i - a_i)z_i \leq A - a_i, \quad i = 1, \dots, n, \tag{1}$$

$$y_i \geq 0, y_i + (a_i - b_i)z_i \leq B - b_i, \quad i = 1, \dots, n, \tag{2}$$

$$-x_i + x_j - (b_i - a_i)z_i + At_{ij} + As_{ij} \geq a_i, \quad i = 1, \dots, n-1, \quad j = i+1, \dots, n, \tag{3}$$

$$x_i - x_j - (b_j - a_j)z_j + At_{ij} - As_{ij} \geq a_j - A, \quad i = 1, \dots, n-1, \quad j = i+1, \dots, n, \tag{4}$$

$$-y_i + y_j - (a_i - b_i)z_i - Bt_{ij} + Bs_{ij} \geq b_i - B, \quad i = 1, \dots, n-1, \quad j = i+1, \dots, n, \tag{5}$$

$$y_i - y_j - (a_j - b_j)z_j - Bt_{ij} - Bs_{ij} \geq b_j - 2B. \quad i = 1, \dots, n-1, \quad j = i+1, \dots, n, \tag{6}$$

$$z_i \in \{0, 1\}, \quad i = 1, \dots, n, \tag{7}$$

$$s_{ij} \in \{0, 1\}, \quad i = 1, \dots, n-1, \quad j = i+1, \dots, n. \tag{8}$$

$$t_{ij} \in \{0, 1\}, \quad i = 1, \dots, n-1, \quad j = i+1, \dots, n. \tag{9}$$

Restrictions (1), (2) define the conditions of accommodation rectangles on the sheet and restrictions (3)–(6) define the conditions pairwise no intersections rectangles. Detail conclusion of the model (1)–(9) is given in [1].

If to add to the model (1)–(9) objective zero function to find its solution, we can use the Land and Doig method, which is the branches and borders method and is intended for solving the partially integer linear programming problem. Stop calculations made after receiving the first valid solution.

Numerical experiments showed practical applicability of the model for solving problems with count of rectangles less 15.

REFERENCES

1. A.A.Andrianova, T.M.Mukhtarova, V.R.Fazylov. *Models of nonguillotine sheet and strip rectangular packing problem.* — Uchenye Zapiski Kazanskogo Universiteta. Seriya Fiziko-Matematicheskie Nauki, 2013, vol. 155, no. 2, pp. 5-18.