ON THE ASYMPTOTIC OPTIMALITY OF SOLVING THE MAXIMUM-WEIGHT m-PERIPATETIC SALESMAN PROBLEM[1]

E. Kh. Gimadi, O. Yu. Tsidulko

Sobolev Institute of Mathematics SB RAS, Novosibirsk State University, Novosibirsk
e-mail: gimadi@math.nsc.ru, tsidulko.ox@gmail.com

The maximum-weight m-peripatetic salesman problem (m-PSP) can be formulated as follows. Given a complete undirected weighted graph $G = (V, E)$, the problem is to find m edge-disjoint Hamiltonian cycles $H_1, \ldots, H_m \subset E$, that maximize the sum

$$W(H_1, \ldots, H_m) = \sum_{k=1}^{m} \sum_{e \in H_k} w(e),$$

where $w : E \to R^+$ is a weight function for the edges in G. If $m = 1$, the problem is the well-known traveling salesmen problem.

The m-PSP is NP-hard in case of arbitrary weight function, as well as in case of Euclidean weight function [1]. This fact encourages development of approximation algorithms for the m-PSP with some guaranteed performance ratios.

Earlier in [2] the m-PSP$_{max}$ in a multi-dimensional Euclidean space was studied. An approximation algorithm with the cubic time complexity was constructed for this problem and the restriction on the number of salesmen m under which the algorithm is asymptotically exact was presented.

Since in most applied problems the input data are represented by real numbers with fixed number of digit positions, and taking into account the scalability of m-PSP, it is of interest to consider the special case of the Euclidean m-PSP$_{max}$ where all the vertices of graph G belong to the set of points of an integer lattice. For this problem in the present paper we construct an approximation polynomial algorithm, based on the ideas from [2] and [3]. The time complexity of the algorithm is $O(n^3)$. The conditions of asymptotic exactness of the algorithm that depend on the diameter of the input graph are obtained. For instances of the problem considered we have identified wide subclasses for which the new algorithm has better approximation ratios than the algorithm from [2].

REFERENCES

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