SEPARABILITY PROBLEMS OF TWO SETS BY PIECEWISE LINEAR AND QUADRATIC FUNCTIONS AS OPTIMIZATION PROBLEMS[1]

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The separability problem of the two finite sets arising in classification problems, formulated as follows [1]. Let there be given two finite sets $\mathcal{A} = \{a^1, \ldots, a^M\}$ and $\mathcal{B} = \{b^1, \ldots, b^N\}$ in the space $I\!R^n$. The separability problem of the sets $\mathcal{A}$ and $\mathcal{B}$ consists in finding a discriminant function $F(\cdot)$ such that $F(x) > 0 \ \ \forall x \in \mathcal{A}, \quad F(x) < 0 \ \ \forall x \in \mathcal{B}$.

The classical problem of linear separation [2] consists in constructing a hyperplane $H(\omega, \xi) = \{x \in I\!R^n | \ \langle \omega, x \rangle = \xi\}$ such that $\langle \omega, x \rangle > \xi \ \forall x \in \mathcal{A}, \ \langle \omega, x \rangle < \xi \ \forall x \in \mathcal{B}$. A linear separation problem is equivalent to a linear programming problem. Unfortunately, in practical problems linear separability often does not take place.

Various definitions of generalized separability have been proposed to satisfy the needs of applications. The first one is bilinear separability. The problem of bilinear separability consists in finding two separating hyperplanes. It is equivalent to nonconvex bilinear optimization problem [3]. The next step of the generalization is the problem of searching for p hyperplanes that separate sets $\mathcal{A}$ and $\mathcal{B}$. In this case we deal with polyhedral separability problem [4] which is equivalent to nonconvex problem with nondifferentiable functions.

Along with piecewise linear separating functions we consider the separability problems with quadratic discriminant functions $F(\cdot)$. Such problems consist in finding sphere or ellipse separating the two sets [5] and these problems are also reduced to nonconvex problems with nondifferentiable functions.

For solving the nonconvex problems we used the global search theory developed for d.c. minimization problems [6]. The algorithms elaborated on this basis are tested on specially constructed test examples and on classification problems from http://archive.ics.uci.edu/ml/datasets.html.

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