

# THE NP-HARD PROJECT MANAGEMENT PROBLEM WITH CREDITS<sup>1</sup>

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The project management problem with reinvestment of incomes and an opportunity to use credits is considered. There is a set of jobs  $V = \{1, 2, \dots, n\}$  and partial order of their execution  $E$ . Each job  $j \in V$  is characterized by the duration  $p_j$  and the cash-flow  $c_j(\tau)$ ,  $\tau = 0, 1, \dots, p_j$ . There are financial resources  $K(t)$ ,  $t = 1, 2, \dots, T$ , where  $T$  is the planning period. The discount rate  $r_0$  is given.

In this paper we research the model from [1], in which there is an opportunity to use credit in every moment of time. The credit rate is  $r_k \geq r_0$ . Let  $D(t)$  be the credit size in the year  $t$ . The model with credits and reinvestment of incomes has the following form: create a schedule of jobs execution  $S = (s_1, s_2, \dots, s_n)$ , where the jobs execution technological order is kept  $s_i + p_i \leq s_j$ ,  $(i, j) \in E$ , and in every moment of time  $t^* = 1, 2, \dots, T$  the positive balance of payments with credits is saved:

$$\sum_{t=1}^{t^*} \left( \frac{K(t)}{(1+r_0)^{t-1}} + \sum_{j \in N_t} \frac{c_j(t-s_j)}{(1+r_0)^{t-1}} + \frac{D(t) - (1+r)D(t-1)}{(1+r_0)^{t-1}} \right) \geq 0,$$

where  $N_t = \{j \in V \mid s_j \leq t < s_j + p_j\}$  is a set of jobs which are performed in the interval  $[t, t+1)$ . The purpose is to maximize the net present value including credits:

$$NPV(S) = \sum_{j \in V} \sum_{\tau=0}^{p_j} \frac{c_j(\tau)}{(1+r_0)^{s_j+\tau}} + \sum_{t=1}^T \frac{D(t) - (1+r)D(t-1)}{(1+r_0)^{t-1}} \rightarrow \max.$$

In this paper complexity of the problem with credits was researched and the following theorem was proved.

**Theorem.** *The project management problem with credits is NP-hard in the strong sense.*  
There is given an example that shows the necessity of credits optimization.

## ЛИТЕРАТУРА

1. E.A. Martynova, V.V. Servakh *On scheduling credited problem.* — Automation and Remote Control. — 2012, Vol. 73, Is. 3, p. 508-516.

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