ON THE ORDER OF SINGULARITY OF THE GENERALIZED SOLUTION OF THE VOLTERRA INTEGRAL EQUATION OF CONVOLUTIONAL TYPE IN BANACH SPACES[1]

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Let us denote E_1, E_2 are real Banach spaces, $\mathbf{u} = u(t)$, $\mathbf{f} = f(t)$ are functions of nonnegative real argument t and with values in E_1 and E_2 respectively. Let us consider integral equation

$$B\mathbf{u} - g * A\mathbf{u} = \mathbf{f}. \tag{1}$$

Here B and A are closed linear operators from E_1 to E_2 at that $\overline{D(B)} = \overline{D(A)} = E_1$ and $D(B) \subseteq D(A)$, kernel $g(t)$ is numerical function. It is assumed that the operator B is Fredholm, i. e. $\overline{R(B)} = R(B)$ and $\dim N(B) = \dim N(B^*) = n < +\infty$, and the function $g(t)$ has a null of order r at $t = 0$.

For example, next boundary value problem can be reduced to this integral equation

$$(\Delta - \alpha)\varphi(t, \bar{x}) + \int\limits_0^t (t-\tau)\beta\varphi_{x_N^2}(\tau, \bar{x})d\tau = f(t, \bar{x}),\ t > 0,\ \bar{x} = (x_1, \ldots, x_N) \in \Omega;\ \varphi(t, \bar{x})|_{\bar{x}\in\partial\Omega} = 0,$$

where Ω is domain of $\mathbb{R}^N$ with boundary $\partial\Omega$ of C^∞ class, constants α and β are not equal to zero. In the case of $N = 3$, this boundary value problem describes the low-frequency electronic (ionic) magneto-acoustic oscillations in an external magnetic field [1].

Using the method of fundamental operator-function of degenerated integro-differential operator in Banach spaces [2] we have constructed generalized solution of integral equation (1) in the class of distributions with left-bounded support proved its uniqueness. We have shown the connectivity between order of generalized solution singularity [3] and order r of function $g(t)$ null at point $t = 0$. Also we have obtained the conditions at which the order of singularity of the generalized solution is equal to zero, it means that generalized solution coincides with the continuous (classical) solution of integral equation (1). Abstract results are illustrated by the example of boundary value problem in plasma physics.

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