A few algorithms for some quadratic Euclidean problems of choosing vector subset and subsequence[1]

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We consider following strongly NP-hard [1]

Problem. *Given a sequence* $\mathcal{Y} = (y_1, \ldots, y_N)$ *of vectors from* $\mathbb{R}^q$*, and positive integer numbers* $M > 1$*,* $T_{\min}$ *and* $T_{\max}$*. Find a subset* $\mathcal{M} = \{n_1, \ldots, n_M\} \subseteq \{1, \ldots, N\}$ *such that*

$$\sum_{n \in \mathcal{M}} \|y_n - \overline{y}(\mathcal{M})\|^2 \to \min,$$

where $\overline{y}(\mathcal{M}) = \frac{1}{|\mathcal{M}|} \sum_{i \in \mathcal{M}} y_i$*, under constraints*

$$1 \le T_{\min} \le n_m - n_{m-1} \le T_{\max} \le N, \ m = 2, \ldots, M,$$

on the elements of $\mathcal{M}$*.*

In the case of $T_{\min} = 1$ and $T_{\max} = N$ the formulated problem is equivalent to one strongly NP-hard problem [2] of choosing «compact» subset in $\mathcal{Y}$.

In this work we present several algorithms for considered problems.

For the problem of choosing a vector subset we have proposed:

– 2-approximation algorithm having a $\mathcal{O}(qN^2)$-time complexity;

– an exact pseudo-polynomial algorithm with $\mathcal{O}(qN(2MB)^q)$-time complexity for the case when the dimension q of space is fixed and the vectors have integer components, here B is maximal absolute value of input vectors components;

– fully polynomial time approximation scheme (FPTAS) for the case when the dimension q of space is fixed; the presented algorithm builds a $(1+\varepsilon)$-approximate solution in $\mathcal{O}(N^2(M/\varepsilon)^q)$-time.

For the problem of choosing a vector subsequence we have proposed:

– 2-approximation algorithm with $\mathcal{O}(N^2(q + N^2))$-time complexity;

– an exact pseudo-polynomial algorithm with $\mathcal{O}(N(q + N^2)(2MB)^q)$–time complexity for the case when the dimension q of space is fixed and the vectors have integer components.

REFERENCES

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