OPTIMALITY CONDITIONS FOR IMPULSIVE CONTROL PROBLEMS WITH INTERMEDIATE STATE CONSTRAINTS[1]

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This report deals with impulsive optimal control problems with trajectories of bounded variation and impulsive controls (regular vector measures). The problem under consideration is characterized by two main features. First, the dynamical control system is linear in the impulsive control and does not possess the so-called well-posedness property. Second, there are intermediate state constraints on the one-sided limits of the trajectory of bounded variation.

An impulsive control system of the following form

$$dx(t) = f(t, x(t), u(t))dt + G(t, x(t))\pi(\mu), \quad u(t) \in U \text{ a.e. on } T, \quad \pi(\mu) \in \mathcal{W}(T, K) \tag{1}$$

is considered. Here, $T = [a, b]$ is a time interval, U is a compact set in R^r, K is a convex closed cone in R^m, $u(\cdot)$ and $\pi(\mu)$ are an ordinary control and an impulsive control correspondingly, and $x(\cdot) \in BV(T, R^n)$, where $BV(T, R^n)$ is the Banach space of R^n-valued functions of bounded variation on T. The set of the impulsive controls $\mathcal{W}(T, K)$ consists of all pairs $(\mu, \gamma(\mu))\}$ such that μ is a K-valued bounded Borel measure on T and $\gamma(\mu)$ is the set $\{d_s, \omega_s(\cdot)\}_{s\in S}$ satisfying the following conditions:

(a) $S \supseteq S_d(\mu) := \{s \in T \mid \mu(\{s\}) \neq 0\}$, S is at most denumerable set in T;

(b) $\forall\ s \in S \quad d_s \geq 0,\ \ \omega_s : [0, d_s] \to co\ K_1, \quad d_s \geq ||\mu(\{s\})||, \quad \int_0^{d_s} \omega_s(\tau)d\tau = \mu(\{s\})$;

(c) $\quad \sum_{s\in S} d_s < \infty.$

Here, $K_1 = \{v \in K \mid ||v|| = 1\}$ and $||v|| = \sum_{j=1}^m |v_j|$. The solution concept of (1) we adopt with some modifications was stated in [1].

In this report, necessary and sufficient optimality conditions that correspond to the Hamilton–Jacobi canonical optimality theory [2, 3] and involve some sets of Lyapunov type functions are presented and discussed. These functions are strongly or weakly monotone solutions of the corresponding Hamilton-Jacobi inequalities and may be compound, i.e. defined piecewise in the variable t. Some examples illustrating the optimality conditions are discussed.

REFERENCES

1. B.M. Miller, E.Ya. Rubinovich *Impulsive controls in continuous and discrete-continuous systems.* — Kluwer Academic Publishers, New York, 2003.
2. V. Dykhta, O. Samsonyuk *Some applications of Hamilton-Jacobi inequalities for classical and impulsive optimal control problems.* — European Journal of Control, vol. 17, 2011, pp. 55–69.
3. O.N. Samsonyuk *Compound Lyapunov type functions in control problems of impulsive dynamical systems.* — Trudy Inst. Matem. i Mekhan., URO RAN, Ekaterinburg, vol. 16, N 5, 2010, pp. 170–178.

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