HEURISTICS FOR MINIMUM SPANNING K-TREE PROBLEM[1]

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We consider a NP-hard combinatorial optimization problem of finding a spanning k-tree [1] of minimum weight in a complete weighted graph, known in literature as *Minimum Spanning k-Tree Problem* (MSkT). MSkT has a number of applications in designing reliable telecommunication networks [2] and generalizes a classical problem in graphs, the *Minimum Spanning Tree Problem* [3].

DEFINITION. [1] *A k-tree is a member of a class of undirected graphs defined recursively as follows: complete graph with k vertices is a k-tree; if T is a k-tree with n vertices, then a new k-tree with $n+1$ vertices is formed by creating a new vertex v and adding edges between v and every vertex of an existing k-clique in T.*

The mathematical formulation of the MSkT is as follows. Let $G = (V, E)$ be a complete weighted undirected graph, where V is a set of nodes and E is the set of edges, and for each edge $[i, j] \in E$ the weight $w(i, j) \geq 0$ is given. Let $T(G)$ be a set of all spanning k-trees in a graph G, where a spanning k-tree is a k-tree that contains all the vertices and a subset of the edges of a graph G. Let $w(T)$ be a weight of edges of the spanning k-tree $T \in T(G)$. It is required to find a spanning k-tree T^* of minimum weight in a complete weighted graph G: $T^* = \arg\min_{T\in T(G)}\{w(T)\}$.

We propose effective heuristics are based on the idea of a well-known Prim's algorithm and based on a dynamic programming approach. We also propose metaheuristics: ant colony algorithm, variable neighborhood search algorithm and genetic algorithm. Preliminary numerical experiment was performed to compare the effectiveness of the proposed algorithms with known heuristics and exact algorithms.

REFERENCES

1. Rose D. *On simple characterizations of k-trees.* Discrete Mathematics, 1974, V. 41, 317—322.
2. Farley A. *Networks immune to isolated failures.* Networks, 1981, V. 11, 255—268.
3. Prim R. *Shortest connection networks and some generalizations.* Bell Systems Techn. J., 1957, V. 36, 1389—1401.

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