Some approximate algorithms for NP-complete problem of sequencing

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For strongly NP-complete scheduling problem - minimizing maximum lateness some properties of optimal permutations is proved and on their basis approximate algorithms for the solution of the problem is developed.

n jobs have to be processed on a single machine not earlier than the time t. Machine can process at most one job at a time and preemption in the process of any job is not allowed. Assume that the jobs are specified numbers from 1 to n. Assume $N = \{1, 2, ..., n\}$. The following data spesified for each job j, $j \in N$: a release date r_j; a processing time $p_j \geq 0$; a due date d_j. Numbers t, r_j, p_j, d_j are integer. We will understand a permutation of all elements the set of jobs N as the schedule. We will denote $\Pi(N, t)$ the set of schedules on the set of jobs N which are begining at time t. The problem consists in searching of such schedule π^* that $\max\limits_{j \in N}\{t_j(\pi^*) - d_j\} = \min\limits_{\pi \in \Pi(N,t)} \max\limits_{j \in N}\{t_j(\pi) - d_l\}$, where $t_j(\pi)$ is completion time job j in the schedule π. Schedule π^* is called an optimal.

Let $N' \subseteq N$, $N' \neq \oslash$, $t' \geq t$, the j_d is job with smallest due date from N', $\pi \in \Pi(N', t')$.

Assume $J_{max}(\pi, j_d) = \{j \in N' \setminus \{j_d\} : r_j \geq r_{j_d}, j_d \xrightarrow{\pi} j\}$, $J_{min}(\pi, j_d) = \{j \in N' \setminus \{j_d\} : r_j < r_{j_d}, j_d \xrightarrow{\pi} j\}$. We note that the sets $J_{max}(\pi, j_d)$, $J_{min}(\pi, j_d)$ can be empty. In paper [1] existenceis optimal schedule π^* that $J_{max}(\pi^*, j_d) = \{j \in N' \setminus \{j_d\} : r_j \geq r_{j_d}\}$ is proved and a general scheme for finding this schedule is constructed assuming that the set $J_{min}(\pi^*, j_d)$ can be founded some algorithm A complexity $O(x(n))$, where $x(n)$ is function which dependent from dimension of the problem.

Scheme [1]. Let assume $t_1 = \max\{r_{min}(N), t\}$, $N_1 = N$, $\pi_1 = \pi^{\oslash}$. Let known t_k, N_k, π_k, и $k \geq 1$. If $N_k = \oslash$ then π_k is optimal schedule and process is complete. Otherwise, we choose the job j_d^k with smallest due date from N_k, $J_{max}^k = \{j \in N_k \setminus \{j_d^k\} : r_j \geq r_{j_d^k}\}$, some algorithm A is finding the set $J_{min}^k = J_{min}(\pi^*, j_d^k)$ and assume $N_{k+1} = J_{min}^k \cup J_{max}^k$, $N^k = N_k \setminus (J_{max}^k \cup J_{min}^k \cup \{j_d^k\})$, $\pi_{k+1} = (\pi_k, \vec{\pi}_r^k, j_d^k)$, where $\vec{\pi}_r^k \in \vec{\Pi}_r(N^k, t_k)$, $t_{k+1} = T(\pi_{k+1})$.

Some properties of optimal schedules are proved and on their basis pseudopolynomial algorithms for finding sets J_{min}^k in the realization of the scheme are developed. The number of iterations of algorithms less than n. At each iteration of the first algorithm the job with a biggest release date from not sequence jobs is add to schedule which constructed on the previous step on the place determine as special way. At each iteration of the second algorithm the job with a smalest release date from not sequence jobs is sequenced, structure schedules which constructed on the previous step is partially saved .

ЛИТЕРАТУРА

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