# THE POLYTOPE OF THE GRAPH APPROXIMATION PROBLEM

R.Y. Simanchev, I.V. Urazova

*OSC KNIORP SB RAS, Omsk state university, Omsk*
*e-mail: osiman@rambler.ru, urazovainn@mail.ru*

In this paper we consider the following problem. Let $K_n = (V, E)$ is a complete graph on $n$ vertices without loops and multiple edges, $G$ - subgraph of $K_n$. Denote by $\mathcal{M}(V)$ the family of all subgraphs of $K_n$, which connected components are cliques.The graph approximation problem is to find a graph $M = \mathcal{M}(V)$ that minimizes the functional $\rho(G, M) = |EG \cup EM| - |EG \cap EM|$.

This problem is $NP$-hard In the general case [1]. For this problem, some known polynomially solvable cases [2], constructed estimates of the objective function [3], developed approximate algorithms [4].

We consider the polyhedral structure of the graph approximation problem. The vector incidence of subgraph $H \subset K_n$ is a vector $x^H \in R^E$, where $R^E$ - space associated with the set $E$, with coordinates $x^H_e = 1$ if $e \in EH$ and $x^H_e = 0$ if $e \notin EH$. Correspondingly, the polytope of this problem is a set $P = conv\{x^M \in R^E | M \in \mathcal{M}(V)\}$.

***Theorem 1.*** *Polytope $P$ is the convex hull of integer solutions of the system*

$$\begin{aligned} x_{uv} + x_{u\omega} - x_{v\omega} &\le 1 \\ x_{uv} - x_{u\omega} + x_{v\omega} &\le 1 \\ -x_{uv} + x_{u\omega} + x_{v\omega} &\le 1, \\ x_{uv} &\ge 0, \end{aligned} \tag{1}$$

*where $u, v, \omega \in V$ are possible triples of distinct vertices.*

In these terms, the objective function $\rho(G, M)$ for a given $G$ will have the form

$$f(x) = |EG| + \sum_{e \in E\bar{G}} x_e - \sum_{e \in EG} x_e.$$

***Theorem 2.*** *Each constraint of the system (1) defines a facet of the polytope $P$.*

In addition, the new classes of facet inequalities of $P$ are describes in this paper. For some classes are solving the separation problem.

## REFERENCES

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