NUMERICAL METHOD FOR SOLVING AN INVERSE BOUNDARY VALUE PROBLEM OF HEAT CONDUCTION USING THE VOLTERRA EQUATIONS OF THE FIRST KIND[1]

S.V. Solodusha

Melentiev Energy Systems Institute SB RAS, Irkutsk
e-mail: solodusha@isem.sei.irk.ru

In the report I consider Volterra equation of the first kind

$$\int_0^t K_N(t-s)\,\phi(s)\,ds = g_\delta(t),\ 0 \le s \le t \le T, \tag{1}$$

where

$$K_N(t-s) = 2\pi^2 \sum_{p=1}^{N} (-1)^{p+1} p^2 e^{-\pi^2 p^2 (t-s)}, \tag{2}$$

which is reduced the solution of an inverse boundary value problem of heat conduction [1]. The right side of (1) is an approximation of function $g(t)$ for $\delta > 0$, so that $\forall t \ge 0$ we have $\|g_\delta(t) - g(t)\| \le \delta$.

To illustrate the specifics of Volterra integral equation of the first kind (1), (2) I present the numerical characteristics of the Volterra kernels $K_N \in C_\Delta$, $\Delta = \{t, s/0 \le s \le t \le T\}$ for fixed values N. The table contains the values K_N for $t = 0$, and the roots t^* of equations $K_N(t) = 0$, $N = \overline{10, 21}$, obtained with single precision.

N	t^*	$K_N(0)$	N	t^*	$K_N(0)$
10	0.01378	-1085.656	16	0.00913	-2684.532
11	0.01221	1302.788	17	0.00631	3020.099
12	0.01173	-1539.658	18	0.00809	-3375.405
13	0.01022	1796.268	19	0.00516	3750.449
14	0.01019	-2072.62	20	0.00735	-4145.234
15	0.00789	2368.705	21	0.00429	4559.757

I constructed an algorithms for the numerical solution of Volterra equation of the first kind (1), (2) based on the self-regularizing property of discretization procedure. Middle rectangles method and product integration method used as a "base". I found the parameters that define the discretization step. I performed a series of test calculations to test the effectiveness of the numerical method. Computing experiment showed that the difference methods converge on the grid step with the order $\mathcal{O}(h^2)$ in the absence of perturbations of the initial data.

REFERENCES

1. Yaparova N.M. *Numerical Methods for Solving a Boundary Value Inverse Heat Conduction Problem*// Inverse Problems in Science and Engineering, 2013, http://www.tandfonline.com/doi/abs/10.1080/17415977.2013.830614

[1]The work was supported by RFBR (grant N 12-01-00722-a).