ON FEEDBACK NECESSARY OPTIMALITY CONDITIONS FOR LINEAR IMPULSIVE CONTROL PROBLEMS [1]

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The talk is concerned with some applications of modern techniques of weakly monotone solutions to Hamilton-Jacobi inequalities for a class of nonconvex optimal impulsive control problems with trajectories of bounded variation. The system's dynamics is described by a measure differential equation, which is linear with respect to a state variable. We discuss nonstandard duality and nonlocal variational necessary optimality conditions with feedback descent controls [1].

Consider the following optimal control problem (P): Minimize $I = \langle c, x(1)\rangle$ subject to the dynamical system

$$dx = [A(t,u)x + a(t,u)]\ dt + [B(t,u)x + b(t,u)]\ \vartheta(dt), \quad x(0-) = x_0.$$

Here, $x(t) \in R^n$, usual control u is a Borel measurable function with values in a given compact set $U \subset R^r$, and ϑ is impulsive control in the sense [2], which is a certain family of Borel measures and measurable functions. The main component of ϑ (the actual impulsive control) is a scalar Borel measure μ. The system is subject to an additional constraint on the total impulse of control: $\nu([0,T]) \leq M$ with a given $M > 0$, where a scalar nonnegative measure ν (another component of ϑ) is a majorant of the total variation of μ.

For nonconvex problem (P) we establish the nonstandard duality: We prove that (P) is an equivalent of a certain comparison problem, which is formulated in terms of adjoint trajectories from the Maximum Principle [3]. Admissible sets of these problems, their minimizing sequences and values are shown to coincide. By means of the well-known time reparameterization technique [3], problem (P) is transformed to an equivalent conventional optimal control problem (RP) with absolutely continuous trajectories. In the reduced problem, we face a terminal constraint imposed on the component of state trajectory, that corresponds to time of the original system. For problem (RP) we obtain variational necessary optimality conditions within Pontryagin formalizm, but with the use of feedback controls [1]. Unfortunately, the discussed result is not yet completely translated into the original impulsive framework. Still, it is effectively used for design of numerical algorithms for optimal impulsive control.

REFERENCES

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