ON EXISTENCE OF CORE IMPUTATIONS IN SOME CLASSES OF FUZZY TU-COOPERATIVE GAMES[1].

V.A. Vasil'ev

Sobolev Institute of Mathematics, Novosibirsk
e-mail: vasilev@math.nsc.ru

In the paper, we analyze so-called $\mathcal{S}$-representation of fuzzy TU-cooperative game. By applying this representation we facilitate essentially investigation of the core imputation existence problem. Special attention is paid to the games with rather simple structure of their $\mathcal{S}$-representation. In particular, some new analogs of superlinearity are obtained that provide nonemptyness of the core for the continuous homogeneous fuzzy games.

Introduced by the author, $\mathcal{S}$-representation v^* of a fuzzy n-person TU-cooperative game $v: [0,1]^n \to \mathbf{R}$ is defined by the formula

$$v^*(\tau^*) := \sup\{v(t\tau^*)/t \mid t \in (0, 1/\parallel \tau^* \parallel_\infty]\}, \quad \tau^* \in \mathcal{S},$$

with $\mathcal{S}$ to be a standard simplex $\mathcal{S} := \{\tau \in [0,1]^n \mid \sum_{i=1}^n \tau_i = 1\}$, and

$$\parallel \tau \parallel_\infty := \max\{|\tau_i| \mid i = 1, \ldots, n\} \quad \tau = (\tau_1, \ldots, \tau_n) \in \mathbf{R}^n$$

(for more details, see [1,2]). One of the main results of the paper is the following criterium of nonemptyness of the core.

Theorem. *Let v be a continuous and homogeneous fuzzy n-person TU-cooperative game. Then there exists a core imputation of this game if and only if for each $g \in \mathbf{R}^n$ such that $\sum_{i=1}^n g_i = 0$ and $\parallel g \parallel_\infty \leq 1/n$ it holds*

$$v(c^*) \geq \frac{1}{2}[v(c^* + g) + v(c^* - g)],$$

with $c^* = (1/n, \ldots, 1/n)$ to be a barycenter of $\mathcal{S}$.

In conclusion, we apply the results obtained to the core imputation existence problem for several types of fuzzy cooperative games arising from concrete game-theoretic settings (airport games, linear production games, etc.)

REFERENCES

1. J.-P. Aubin *Optima and Equilibria.* Berlin-Heidelberg: Springer-Verlag, 1993. — 417p.
2. V.A. Vasil'ev *An extension of Bondareva-Shapley theorem to fuzzy cooperative games.* — VII Moscow International Conference on Operation Research (ORM2013): Moscow, October 15-19, 2013: Proceedings: Vol.I. — Moscow, MAKS Press, 2013. — p.209-212.

[1]Supported by the Russian Fund for Basic Research (Grant N13-06-00311) and Russian Fund for Humanitarian Research (Grant N13-02-00226)