ABOUT THE BRANCH AND BOUND METHOD IN OPTIMIZATION PROBLEMS WITH INTERVAL UNCERTAINTY

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The general approach within the bounds of the branch and bound method (BBM) to solve the minimization problem in the interval formulation is substantiated in the report.

Let it be a functional $F(x)$, defined on the set X $(x \in X)$ - centered intervals; $F(x) \in X$, i.e. value that it takes, also let it be an element of the set of centered intervals. Let $D \subset X$ - is an admissible set of centered. intervals.

We introduce a linear order on the set $A = \{a_1, ..., a_k\}$ centered intervals $a_i = (\alpha_i, \sigma_i)$, $i \in J_k = \{1, 2, ..., k\}$. We introduce characteristic comparators for the centered intervals $a_i = (\alpha, \sigma)$: $(\alpha - \sigma, \alpha + \sigma) \subset R^1$, $\sigma \geq 0$:

1) $H_1 = (\alpha, \sigma) = \sqrt{\alpha^2 + \sigma^2} sign(\alpha)$.

2) $H_2 = (\alpha, \sigma) = (|\alpha| + \sigma) sign(\alpha) = \begin{cases} \alpha - \sigma, \ \alpha > 0 \\ \alpha + \sigma, \ \alpha < 0 \end{cases}$, if $H_1(\alpha, \sigma) = H_1(\beta, \delta)$. Here $sign(\alpha) = 1, alpha > 0$; $sign(\alpha) = 0, alpha = 0$; $sign(\alpha) = -1, alpha < 0$.

3) If $H_t(a_i) = H_t(a_j)$, $t = 1, 2$, that $\alpha_i \neq 0$, $H_3(a_i) = \alpha_i$, $i \in J_k$.

Such comparator we call H and denote$H =< H_1, H_2, H_3 >$.

The binary relation of order $\prec$ between intervals a_i, a_j, $i, j \in J_k$, we set as following.

1) If $H_1(a_i) < H_1(a_j)$, then $a_i \prec a_j$.

2) If $H_1(a_i) < H_1(a_j)$, $H_2(a_i) = H_2(a_j)$, then $a_i \prec a_j$.

3) If $H_1(a_i) = H_1(a_j)$, $H_2(a_i) = H_2(a_j)$, then: or a) $a_i = a_j$ $H_3(a_i) = H_3(a_j)$, say by definition: $a_i \prec a_j$ (or $a_j \prec a_i$), because $a_i = a_j$; or b) $a_i \neq a_j$, $a_i = (\alpha_i, \sigma_i)$, $a_j = (\alpha_j, \sigma_j)$ and $|\alpha_i| = \sigma_j \neq 0$, $|\sigma_i| = \alpha_j \neq 0$ (in this case $\alpha_i \neq \alpha_j$, $\sigma_i \neq \sigma_j$, $H_3(a_i) = \alpha_i$, say then $a_i \prec a_j$, if $H_3(a_i) < H_3(a_j)$; or c) $a_i \neq a_j$, $\alpha_i = \alpha_j = 0$, $\sigma_i \neq \sigma_j$, then $H_3(a_i) = \sigma_i$ and $H_3(a_j) = \sigma_j$, if $H_3(a_i) < H_3(a_j)$, that $a_i \prec a_j$.

Theorem. *The binary relation $\prec$ between the centered intervals, which is given by the comparator $H =< H_1, H_2, H_3 >$, is the linear order.*

Let $A = \{a_1, ..., a_k\}$, $a_1 \prec a_2 \prec ... \prec a_{k-1} \prec a_k$. We will call maximum a_k: $a_k = \max_{a_i \in A}\{a_i | i = 1, 2, ..., k\}$; minimum - a_1: $a_1 = \min_{a_i \in A}\{a_i | i = 1, 2, ..., k\}$.

Using the operations above centered intervals and definitions of elementary functions [1], the optimization problem on set of centered intervals D can be formulated as the following: find $\min_{x \in D} F(x)$.

In [1] BBM for minimizing of functional on the set of the intervals is proposed and justified. Further numerical experiments are necessary to establish the framework for the practical applying of the method.

REFERENCES

1. Sergienko I. V.,Iemets Ol. O., Yemets' Ol. O. *Optimization Problems with Interval Uncertainty: Branch and Bound Method.* — Cybernetics and Systems Analysis, Vol.49, I.5, 2013, pp. 673-683