BILEVEL OPTIMIZATION: MODEL, OPTIMALITY CONDITIONS AND FIRST SOLUTION APPROACHES [1]

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Considering the convex parametric optimization problem $\varphi(x) := \min_y\{f(x,y) : g(x,y) \leq 0\}$ with the solution set mapping Ψ, the bilevel optimization problem reads as

$$\text{"}\min_x\text{"}\{F(x,y) : G(x) \leq 0,\ y \in \Psi(x)\}. \tag{1}$$

Here $F, f, g_i, i = 1, \ldots, p$, are real functions mapping the $m \times n$-dimensional Euclidean spaces to the real line, and the functions $G_i, i = 1, \ldots, q$, are defined on the n-dimensional real space. First note, that problem (1) is not well posed in case that the set $\Psi(x)$ does not reduce to a singleton for all feasible x. In that case there are at least two possible ways out. Here we will use the optimistic approach $\min_{x,y}\{F(x,y) : G(x) \leq 0,\ y \in \Psi(x)\}$.

To solve this problem, it needs to be replaced with a one-level optimization problem. For that, at least two approaches are possible:

1. The use of the Karush-Kuhn-Tucker (KKT) conditions to replace the lower level problem. This leads to a mathematical program with complementarity constraints (MPCC) which, unfortunately, is not fully equivalent to (1), see Dempe and Dutta [1]. Necessary optimality conditions using this approach can be formulated for the bilevel optimization problem using ideas of nonsmooth analysis, see Zemkoho [2]. First algorithms for computing stationary solutions of problem (1) are presented. These algorithms have to respect that the Lagrange multipliers for the lower level problem need to be carefully selected such that a local optimum of the (MPCC) is really related to a local optimum of problem (1).

2. Replacing the lower level problem using the optimal value function $\varphi(x)$ leads to a nonsmooth (since the function $\varphi(\cdot)$ is not differentiable even under very restrictive assumptions) optimization problem

$$\min_{x,y}\{F(x,y) : G(x) \leq 0,\ g(x,y) \leq 0,\ f(x,y) \leq \varphi(x)\}.$$

   Partial calmness can be used to replace this problem using an exact penalty function approach. For the obtained Lipschitz optimization problem, necessary optimality conditions can be formulated using variational analysis again. Solution algorithms using this approach need to approximate the optimal value function, see e.g. Dempe and Franke [3] in the linear case.

REFERENCES

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