ALGORITHMS FOR QUASIUNIFORM APPROXIMATION OF THE REACHABLE SET OF THE CONTROLLED SYSTEM WITH TWO AND THREE PHASE VARIABLE [1]

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The paper considers the algorithms for the search of accessible points that fills volume of the reachable set in a quasi-uniform way, and thus, unlike the method of stochastic approximation resulting in a cloud of points uniformly (with some accuracy) approximating the set even with a small number of points. The proposed algorithms are similar to the method of "deep holes"[1] and requires multiple solution of auxiliary optimization problems for adding points to the approximating set.

Let's consider nonlinear controlled systems $\dot{x} = f(x(t), u(t), t)$ with the initial conditions $x(t_0) = x_0$ and control constraints $\underline{u} \leq u(t) \leq \overline{u}$ on $t \in [t_0, t_1]$. Algorithms for constructing approximations include sequential addition of points $x^i(t_1)$ to the set $\{x^n\}$ obtained at the previous stages. In the first variant of the algorithm the problem of searching of each additional point consits in solving the maximin optimal control problem $x^* = \max_u \min_{i=\overline{1,n}} \rho_i$, where $\rho_i = ||x^i - x(t_1)||_2$. In order to use optimization methods designed for smooth functionals for solving this problem it was proposed several versions of smooth approximation of discrete maximin functional. An intermediate solution will be obtained after completing each stage of optimization, so elapsed computing time can be viewed as an additional criterion for stop this algorithm. The second algorithm is based on the minimization of a continuous function $x^* = \min_u \sum_{i=1}^{nb} S(\rho_i)$ that depends of the distance between the points and determines to be equal to zero if ρ_i more than the desired threshold value, equal to a sufficiently large number in $\rho_i = 0$, and monotonically decrease in the gap. The fact that the lower bound of optimal value of the functional is known in advance greatly reduces computational time while using stochastic algorithms of global optimization algorithm. The algorithms stops when it is impossible to add a point that satisfies the uniformity condition. If the number of approximation points are the order of 10^2 results of the algorithms are close, but with the successful choice of the function the second algorithm will provide a solution quickly, while, with a smaller number of approximating points, the first algorithm will produce much more reliable results. The boundary points of its resulting set will be the boundary points of the reachable set with an accuracy of solving optimization problems, in particular that is because it's no necessary to setup algorithm parametres. Computational experiments with the proposed algorithms are performed for two-dimensional and three-dimensional problems, the results obtained by visualization of test reachable sets.

REFERENCES

Kamenev G.K. APPROXIMATION OF COMPLETELY BOUNDED SETS BY THE DEEP HOLES METHOD. Computational Mathematics and Mathematical Physics. 41:11 (2001), 1667–1675.

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