APPROXIMATION ALGORITHMS FOR NP-HARD POLYHEDRAL SEPARABILITY PROBLEMS[1]

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We consider monochromatic ball covering problem as a preliminary step to solve NP-hard polyhedral separability problem.

Problem 1. Given set of points $X = \{(x_i, y_i)\}_{i=1}^{l}$ where $x_i \in \mathbb{R}^d$ and $y_i \in \{-1, 1\}$ find the smallest cardinality disjunct cover $\mathcal{D}$ of balls for X such that for every $D \in \mathcal{D}$ we have $y = 1$ for all $z = (x, y) \in X \cap D$ or $y = -1$ for each $z \in X \cap D$.

In problem 1 notations let us formulate problem 2 of polyhedral separability [2] of two subsets A and B of X having y-coordinates $y = 1$ and $y = -1$ respectively. Specifically the problem is to get the smallest cardinality set of hyperplanes such that for each pair of points $a \in A$ and $b \in B$ there is some hyperplane in the set which strictly separates a and b.

It turns out that every feasible solution $\mathcal{D}$ of cardinality N for problem 1 can be easily transformed into the feasible solution for problem 2. For $d = 2$ a power diagram (which is more general than Voronoi diagram) for disks of $\mathcal{D}$ could be obtained in time $O(N \log N)$ from which a feasible solution for problem 2 of cardinality $O(N)$ should be easily extracted in time $O(N)$. For $d > 2$ analogous transform would be done in time $O(N^2)$ with feasible solution of cardinality $O(N^2)$ to the second problem. This transform has the following statistical analogue: given clustered data find a soft power diagram having the largest margin [1].

To estimate the accuracy of the solution for problem 2 thus obtained let us denote by k_{opt} the minimal value for the problem. Also by N_{opt} we mean the smallest cardinality of the partition of X into "monochromatic" blocks (subsets whose points have equal y-coordinates) having convex hulls which contain no points of X with different y-coordinate. Then for $d = 2$ the accuracy of the solution of problem 2 could be expressed in terms of the accuracy for corresponding problem 1 as $O(k_{opt}\Delta)$ where $\Delta = N/N_{opt}$. In view of the tight bound $k_{opt} \geq \Theta(\sqrt[d]{N_{opt}})$ the accuracy would be looser for $d > 2$.

To get an approximate solution for problem 1 we use $O\left(|X|^{\lceil \frac{d}{2} \rceil} (\varepsilon^{-2} \log |X|)^{\lceil (d+1)/2 \rceil}\right)$-procedure for approximate (to accuracy $\varepsilon > 0$) ball search which contains the largest cardinality subset of A (resp. of B) while avoiding points from B (resp. from A) [3].

REFERENCES

1. S. Borgwardt *On soft power diagrams.* — arXiv:1307.3949[cs.LG]. — 2013, 22p.
2. N. Megiddo *On the Complexity of Polyhedral Separability.* — Discrete and Computational Geometry. — 1988, №3, P. 325–337.
3. Aronov B., Har-Peled S. *On approximating the depth and related problems.* — SIAM Journal of Computing. — 2008, Vol. 38. P. 899–921.

[1]Work was financially supported by Ural Branch of RAS (projects № 12-P-1-1016, 12-S-1-1017/1), RFBR (projects № 13-07-00181, 13-01-00210).