ON UNBOUNDED SOLUTIONS OF AN INITIAL VALUE PROBLEM

N.A. Sidorov

Irkutsk State University , Irkutsk
e-mail: sidorovisu@gmail.com

Consider the differential equation

$$B(t)\frac{dx}{dt} = A(t)x(t) + f(t) \tag{1}$$

with the initial condition

$$\lim_{t\to 0} B(t)x(t) = y_0, \tag{2}$$

$B(t),\ A(t),\ f(t)$ - analytic in a neighborhood of zero . It is assumed that $B(0)-$ Fredholm operator, $\{\phi_1, \ldots, \phi_n\}$ - a basis of Ker$B(0)$, $\{\psi_1, \ldots, \psi_n\}$ - a basis of Coker$B(0)$.

$$\mathrm{Ker}B(0) \subseteq \bigcap_{i=0}^{k-1} \mathrm{Ker}B^{(i)}(0), \quad \det[\langle B_{(0)}^{(k)}\phi_i, \psi_j\rangle]_{i,j=\overline{1,n}} \neq 0,$$

$$\det[\lambda\langle B_{(0)}^{(k)}\phi_i, \psi_j\rangle - \langle A_{(0)}^{(k-1)}\phi_i, \psi_j\rangle]_{i,j=\overline{1,n}} \neq 0$$

for $\lambda = -k+1, -k+2, \ldots$.
If $k \geq 2$, then the additional assumption that

$$\mathrm{Ker}B(0) \subseteq \bigcap_{i=0}^{k-2} \mathrm{Ker}A^{(i)}(0),\ ,\ ,\ ,\ ,\det[\langle A_{(0)}^{(k-1)}\phi_i, \psi_j\rangle]_{i,j=\overline{1,n}} \neq 0$$

Then we have

Theorem. *Initial value problem* (1), (2) *in a punctured neighborhood* $0 < |t| < r$ *is in the class of analytic functions of a unique solution. Solution can be represented in the form of a Laurent series with a pole of order* $k-1$.

If $k = 1$, the point $t = 0$ is a removable singularity of solutions and we arrive at the well-known result (see , for example, [1]) .

REFERENCES

1. G.A. Sviridyuk, S.A. Zagrebina *Problems Showalter - Sidorova as a phenomenon of Sobolev-type equations.* Proceedings of ISU. Ser. Mathematics , 2010, v.3 , в„– 1, p. 104-125 .